Strongly (i, j)-Ipre* Continuous Functions in bi-Intuitionistic Topological Spaces

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Abstract: **In this paper, we introduce the concepts of strongly (i, j)- Ipre* continuous functions in bi-intuitionistic topological spaces. We investigate their fundamental properties and examine their relationships with several existing classes of mappings, pairwise strongly intuitionistic continuous and various strongly continuous functions in bi-Intuitionistic Topological Spaces (bi-ITS). Suitable examples are provided to illustrate the introduced concepts in bi-ITS.**

Keywords: **Bi-Intuitionistic topological spaces, (i, j)- Ipre* open set, (i, j)- Ipre* closed set, (i, j)- Ipre* continuous function, (i, j)- Ipre* Irresolute function and strongly (i, j)- Ipre* continuous function.**

1. INTRODUCTION

D. Coker introduced the concept of intuitionistic sets, while J. C. Kelly introduced bitopological spaces in 1963. Later, in 2015, Taha H. Jassim et al. developed the concepts of (i, j)- Ipre open set, (i, j)- semi open set, (i, j)- Iα open set and (i, j)- Iβ open sets in bi-intuitionistic topological spaces. In our earlier studies, the concepts of (i, j)- Ipre* open sets, (i, j)- Ipre* closed sets, (i, j)- Ipre* continuous functions and (i, j)- Ipre* irresolute functions were introduced and their basic properties were investigated. Building upon these results, the present paper focuses on a stronger form of continuity associated with (i, j)- Ipre* open sets.

In this paper, we introduce the concept of strongly (i, j)- Ipre* continuous functions between bi-intuitionistic topological spaces. Fundamental properties and characterizations of these mappings are established, and their relationships with several existing classes of functions, including strongly (i, j)- Iα* continuous and strongly (i, j)- Isemi*-continuous mappings are investigated. Independent examples are provided wherever necessary to demonstrate the distinctions among these classes of functions.

2. PRELIMINARIES

Definition 2.1 [3] Let X be a nonempty fixed set. An intuitionistic set (IS for short) $\Re$ is an object having the form $\Re = \langle X, \Re_1, \Re_2\rangle$ where $\Re_1, \Re_2$ are subsets of X satisfying $\Re_1 \cap \Re_2 = \emptyset$. The set $\Re_1$ is called the set of members of $\Re$, while $\Re_2$ is called set of non-members of $\Re$.

Definition 2.2 [3] An intuitionistic topology (IT) on a nonempty set X is a family T of IS's in X containing $\emptyset$, X and closed under finite intersection and arbitrary union. In this case the pair (X, IT) is called an intuitionistic topological space (ITS) and any IS in IT is known as an intuitionistic open set (IOS) in X, the complement of IOS is called intuitionistic closed set (ICS) in X.

Definition 2.3 [8] Let (X, IT_i, IT_j) be a bi-intuitionistic topological space(bi-ITS) if for each of (X, IT_i) and (X, IT_j) are ITS on X and let $G \subseteq X$ then G is said to be (i, j)-intuitionistic open set ((i, j)-IOS for short) if $G = \mathfrak{R} \cup \mathfrak{H}$ where $\mathfrak{R} \in IT_i$ and $\mathfrak{H} \in IT_j$. The complement of an (i, j)-intuitionistic open set is called an (i, j)-intuitionistic closed set ((i, j)-ICS for short).

Definition 2.4 [5, 6] Let bi-ITS (X, IT_i, IT_j), $\mathfrak{R} \subseteq X$, then $\mathfrak{R}$ is said to be **(i, j)- Intuitionistic Pre* Open Set** (shortly, an ((i, j)- Ipre* open set) if $\mathfrak{R} \subseteq IT_i int(IT_j cl^*(\mathfrak{R}))$. The class of all (i, j)-Ipre*open set in X is notated as (i, j)-IP*O (X, IT_i, IT_j). The complement of (i, j)-Ipre*open set is called (i, j)- Ipre*closed set.

Definition 2.5 [5, 6] Let bi-ITS (X, IT_i, IT_j), $\mathfrak{R} \subseteq X$, then $\mathfrak{R}$ is said to be

- (i, j)- intuitionistic semi*open set (shortly, (i, j)-IS*OS) if $\mathfrak{R} \subseteq (IT_j cl^*(IT_i int(\mathfrak{R}))$.
- (i, j)- intuitionistic α^*open set (shortly, (i, j)-Iα^*OS) if $\mathfrak{R} \subseteq IT_i int^*(IT_j cl(IT_i int^*(\mathfrak{R})))$.
- (i, j)- intuitionistic β^*open set (shortly, (i, j)-Iβ^*OS) if $\mathfrak{R} \subseteq (IT_j cl(IT_i int^*(IT_j cl(\mathfrak{R})))$.
- (j, i)- intuitionistic pre*open set (shortly, an (j, i)-Ipre*open set) if $\mathfrak{R} \subseteq IT_j int(IT_i cl^*(\mathfrak{R}))$.

The complement of (i, j)-IS*OS, (i, j)-Iα^*OS, (i, j)-Iβ^*OS and (j, i)-Ipre*open set is called (i, j)-IS*CS, (i, j)-Iα^*CS, (i, j)-Iβ^*CS and (j, i)-Ipre*closed set.

Theorem 2.6 [5, 6] In a bi-ITS (X, IT_i, IT_j), then

(i) Every IT_i-open set is (i, j)-Ipre*open set and every IT_i-closed set is (i, j)-Ipre*closed set.

(ii) Every (i, j)-Ipre*open set is (i, j)-Ipre open set and every (i, j)-Ipre*closed set is (i, j)-Ipre closed set.

Definition 2.7 [7] Let (X, IT_i, IT_j) and $(Y, I\sigma_i, I\sigma_j)$ be two bi-ITS. Then a function $f: (X, IT_i, IT_j) \to (Y, I\sigma_i, I\sigma_j)$ is called

a) **(i, j)- Intuitionistic Pre* Continuous** ((i, j)- Ipre* cts) if the inverse image of each $I\sigma_i$-open set V in $(Y, I\sigma_i, I\sigma_j)$ is (i, j)- IP*OS in (X, IT_i, IT_j).

b) **(i, j)- Intuitionistic Pre* Irresolute** ((i, j)- Ipre* irres.) if the inverse image of each (i, j)- Ipre*open set V in $(Y, I\sigma_i, I\sigma_j)$ is (i, j)- Ipre*open set in (X, IT_i, IT_j).

Definition 2.8 [7] Let (X, IT_i, IT_j) and $(Y, I\sigma_i, I\sigma_j)$ be two bi-ITS. Then a function $f: (X, IT_i, IT_j) \to (Y, I\sigma_i, I\sigma_j)$ is called:

a) (i, j)- Intuitionistic Semi* Continuous ((i, j)- Isemi* cts) if the inverse image of each $I\sigma_i$-open set V in $(Y, I\sigma_i, I\sigma_j)$ is (i, j)- IS*OS in (X, IT_i, IT_j).

b) (i, j)- Intuitionistic α^* Continuous ((i, j)- Iα^* cts) if the inverse image of each $I\sigma_i$-open set V in $(Y, I\sigma_i, I\sigma_j)$ is (i, j)-Iα^*OS in (X, IT_i, IT_j).

c) (i, j)- Intuitionistic β^* Continuous ((i, j)- Iβ^* cts) if the inverse image of each $I\sigma_i$-open set V in $(Y, I\sigma_i, I\sigma_j)$ is (i, j)-Iβ^*OS in (X, IT_i, IT_j).

d) Then a function $f: (X, IT_i) \to (Y, I\sigma_i)$ and $f: (X, IT_j) \to (Y, I\sigma_j)$ is said to be Pairwise Intuitionistic Continuous (pairwise I-cts) if inverse image of every $I\sigma_i$-open set (resp. $I\sigma_j$-open set) in Y is IT_i-open set (resp. IT_j-open set) in X.

Definition 2.9 [1] A function $f: (X, IT) \to (Y, I\sigma)$ is called strongly intuitionistic continuous function if the preimage of each intuitionistic set A in Y is intuitionistic open and intuitionistic closed in X.

3. STRONGLY (i, j)- IPRE* CONTINUOUS FUNCTION

Definition 3.1 A function $f: (X, IT_i, IT_j) \to (Y, I\sigma_i, I\sigma_j)$ is called a **Strongly (i, j)- Intuitionistic Pre* Continuous** (strongly (i, j)- Ipre* cts) functions if the inverse image of each (i, j)- Ipre* open set in $(Y, I\sigma_i, I\sigma_j)$ is IT_i-open set in (X, IT_i, IT_j).

Equivalently, $f: (X, IT_i, IT_j) \to (Y, I\sigma_i, I\sigma_j)$ is called strongly (i, j)- Ipre* cts function if $f^{-1}(V) \in IT_i$-OS in (X, IT_i, IT_j) for each (i, j)- Ipre* open set V in $(Y, I\sigma_i, I\sigma_j)$.

Example 3.2 Let X = {s, t}, IT_i = {X, ∅, <X, ∅, ∅>, <X, {s}, ∅>, <X, ∅, {t}>, <X, {t}, ∅>}, IT_j = {X, ∅, <X, ∅, {s}>, <X, {s}, ∅>, <X, {t}, {s}>} and Y = {d, m}, $I\sigma_i$ = {Y, ∅, <Y, {d}, ∅>, <Y, ∅, {m}>}, $I\sigma_j$ = {Y, ∅, <Y, {m}, {d}>}. Here, (i, j)-IP*O (Y) = {Y, ∅, <Y, {d}, ∅>, <Y, ∅, {m}>}. Define f: (X, IT_i, IT_j) → (Y, $I\sigma_i, I\sigma_j$) by f(s) = d, f(t) = m. Clearly, f^{-1}(<Y, {d}, ∅>) = <X, {s}, ∅> and f^{-1}(<Y, ∅, {m}>) = <X, ∅, {t}> are IT_i-open set in (X, IT_i, IT_j). Hence, f is strongly (i, j)- Ipre* cts.

Definition 3.3 A function f: (X, IT_i, IT_j) → (Y, $I\sigma_i, I\sigma_j$) be a two bi-ITS. If a map f: (X, IT_i) → (Y, $I\sigma_i$) and f: (X, IT_j) → (Y, $I\sigma_j$) is said to be Pairwise Strongly Intuitionistic Continuous functions if the inverse image of each subset of Y is IT_i-open (resp. IT_j-open) and IT_i-closed (resp. IT_j-closed) in X.

Theorem 3.4 If a map f: (X, IT_i, IT_j) → (Y, $I\sigma_i, I\sigma_j$) is strongly (i, j)- Ipre* cts then it is pairwise intuitionistic continuous.

Proof: Let Q be an $I\sigma_i$-open set in Y. Since each $I\sigma_i$-open set is (i, j)- Ipre* open set, Q is (i, j)- Ipre* open set in Y. Since f is strongly (i, j)- Ipre* cts, f^{-1}(Q) is IT_i-open set in X. Therefore, f is pairwise intuitionistic continuous.

Remark 3.5 The afterwards example shows that the previous theorem converse doesn't have too accurate.

Example 3.6 Let X = {s, t}, IT_i = {X, ∅, <X, ∅, ∅>, <X, {s}, ∅>, <X, ∅, {t}>, <X, {t}, ∅>}, IT_j = {X, ∅, <X, ∅, {s}>, <X, {t}, {s}>} and Y = {s, t}, $I\sigma_i$ = {Y, ∅, <Y, ∅, ∅>, <Y, {t}, ∅>, <Y, ∅, {t}>}, $I\sigma_j$ = {Y, ∅, <Y, {t}, {s}>}. Here, (i, j)- IP*O (Y) = {Y, ∅, <Y, ∅, ∅>, <Y, ∅, {s}>, <Y, ∅, {t}>, <Y, {t}, ∅>, <Y, {t}, {s}>}. Define f: (X, IT_i, IT_j) → (Y, $I\sigma_i, I\sigma_j$) by f(s) = s, f(t) = t. Clearly, f is pairwise intuitionistic continuous. But, f^{-1}(<Y, {t}, {s}>) = <X, {t}, {s}> is not IT_i-open set in X. Hence, f is not strongly (i, j)- Ipre* cts.

Theorem 3.7 If a map f: (X, IT_i, IT_j) → (Y, $I\sigma_i, I\sigma_j$) is pairwise strongly intuitionistic continuous then it is strongly (i, j)- Ipre* cts.

Proof: Assume that f is pairwise strongly intuitionistic continuous. Let V be any (i, j)- Ipre* open set in Y. Since f is pairwise strongly intuitionistic continuous, the inverse image of each subset of Y is IT_i-open set and IT_i-closed set in X. Hence, f^{-1}(V) is IT_i-open set in X. Therefore, f is strongly (i, j)- Ipre* cts.

Remark 3.8 The afterwards example shows that the previous theorem converse doesn't have too accurate.

Example 3.9 Let X = {m, t}, IT_i = {X, ∅, <X, ∅, ∅>, <X, {m}, ∅>, <X, ∅, {t}>, <X, {t}, ∅>}, $IT_i^{\,c}$ = {X, ∅, <X, ∅, ∅>, <X, ∅, {m}>, <X, ∅, {t}>, <X, {t}, ∅>}, IT_j = {X, ∅, <X, ∅, {m}>, <X, {t}, {m}>} and Y = {p, q}, $I\sigma_i$ = {Y, ∅, <Y, {p}, ∅>, <Y, ∅, {q}>}, $I\sigma_j$ = {Y, ∅, <Y, {q}, {p}>}. Here, (i, j)- IP*O (Y) = {Y, ∅, <Y, {p}, ∅>, <Y, ∅, {q}>}. Define f: (X, IT_i, IT_j) → (Y, $I\sigma_i, I\sigma_j$) by f(m) = p, f(t) = q. Clearly, f is strongly (i, j)- Ipre* cts. But, f^{-1}(<Y, {p}, ∅>) = <X, {m}, ∅> is IT_i-open set in X but not IT_i-closed set in X. Hence, f is not pairwise strongly intuitionistic continuous.

Theorem 3.10 A map f: (X, IT_i, IT_j) → (Y, $I\sigma_i, I\sigma_j$) is strongly (i, j)- Ipre* cts if and only if the inverse image of each (i, j)- Ipre* closed set in Y is IT_i-closed set in X.

Proof: Assume that f is strongly (i, j)- Ipre* cts. Let $\mathfrak{R}$ be any (i, j)- Ipre* closed set in Y. Then $\mathfrak{R}^c$ is (i, j)- Ipre* open set in Y. Since f is strongly (i, j)- Ipre* cts, $f^{-1}(\mathfrak{R}^c)$ is IT_i-open set in X. But $f^{-1}(\mathfrak{R}^c) = X - f^{-1}(\mathfrak{R})$ and so $f^{-1}(\mathfrak{R})$ is IT_i-closed set in X.

Conversely, assume that the inverse image of each (i, j)- Ipre* closed set in Y is IT_i-closed set in X. Let $\mathfrak{R}$ is (i, j)- Ipre* open set in Y then $\mathfrak{R}^c$ is (i, j)- Ipre* closed set in Y. Now by the assumption, $f^{-1}(\mathfrak{R}^c)$ is IT_i-closed set in X, but $f^{-1}(\mathfrak{R}^c) = X - f^{-1}(\mathfrak{R})$ and so $f^{-1}(\mathfrak{R})$ is IT_i-open set in X. Therefore, f is strongly (i, j)- Ipre* cts.

Theorem 3.11 A map f: (X, IT_i, IT_j) → (Y, $I\sigma_i, I\sigma_j$) is strongly (i, j)- Ipre* cts then it is (i, j)- Ipre* continuous.

Proof: Assume that f is strongly (i, j)- Ipre* cts. Let $\mathfrak{R}$ be any $I\sigma_i$-open set in Y. By theorem 2.6 (i), $\mathfrak{R}$ is (i, j)- Ipre* open set in Y. Since f is strongly (i, j)- Ipre* cts, $f^{-1}(\mathfrak{R})$ is IT_i-open set in X. Since each IT_i-open set is (i, j)- Ipre* open set, $f^{-1}(\mathfrak{R})$ is (i, j)- Ipre* open set in X. Therefore, f is (i, j)- Ipre* continuous.

Remark 3.12 The afterwards example shows that the previous theorem converse doesn't have too accurate.

Example 3.13 Let X = {λ, μ}, IT_i = {X, ∅, <X, ∅, ∅>, <X, {μ}, ∅>, <X, {μ}, {λ}>}, IT_j = {X, ∅, <X, {λ}, ∅>}. Here, (i, j)-IP*O (X) = {X, ∅, <X, ∅, ∅>, <X, {λ}, ∅>, <X, ∅, {μ}>, <X, {μ}, ∅>, <X, {μ}, {λ}>, <X, {λ}, {μ}>} and Y = {ω, δ}, $I\sigma_i$ =

{Y, ∅, <Y, ∅, ∅>, <Y, {δ}, ∅>, <Y, ∅, {δ}>}, $I\sigma_j$ = {Y, ∅, <Y, {δ}, {ω}>}. Here, (i, j)- IP*O (Y) = {Y, ∅, <Y, ∅, ∅>, <Y, ∅, {ω}>, <Y, ∅, {δ}>, <Y, {δ}, ∅>, <Y, {t}, {ω}>}. Let f: $(X, IT_i, IT_j) \to (Y, I\sigma_i, I\sigma_j)$ be defined as f(λ) = ω, f(μ) = δ then f^{-1}(<Y, {δ}, ∅>) = <X, {μ}, ∅> and f^{-1}(<Y, ∅, {δ}>) = <X, ∅, {μ}>. Clearly, f is (i, j)- Ipre* continuous. But, f^{-1}(<Y, ∅, {ω}>) = <X, ∅, {λ}> and f^{-1}(<Y, ∅, {μ}>) = <X, ∅, {δ}> which is not IT_i-open set in X. Hence, f is not strongly (i, j)- Ipre* cts.

Theorem 3.14 If a map f: $(X, IT_i, IT_j) \to (Y, I\sigma_i, I\sigma_j)$ and g: $(Y, I\sigma_i, I\sigma_j) \to (Z, I\gamma_i, I\gamma_j)$ be any two functions in bi-Intuitionistic topological spaces.

a) If f is strongly (i, j)- Ipre* cts and g is (i, j)- Ipre* irresolute then their composition g ∘ f: $(X, IT_i, IT_j) \to (Z, I\gamma_i, I\gamma_j)$ is strongly (i, j)- Ipre* cts.

b) If f is (i, j)- Ipre* continuous and g is strongly (i, j)- Ipre* cts then their composition g ∘ f: $(X, IT_i, IT_j) \to (Z, I\gamma_i, I\gamma_j)$ is (i, j)- Ipre* irresolute.

c) If f is strongly (i, j)- Ipre* cts and g is (i, j)- Ipre* continuous then their composition g ∘ f: $(X, IT_i, IT_j) \to (Z, I\gamma_i, I\gamma_j)$ is pairwise intuitionistic continuous.

d) If f is pairwise intuitionistic continuous and g is strongly (i, j)- Ipre* cts then their composition g ∘ f: $(X, IT_i, IT_j) \to (Z, I\gamma_i, I\gamma_j)$ is strongly (i, j)- Ipre* cts.

Proof:

a) Let $\Re$ be any (i, j)- Ipre* open set in Z. Since g is (i, j)- Ipre* irresolute, $g^{-1}(\Re)$ is (i, j)- Ipre* open set in Y. Also, f is strongly (i, j)- Ipre* cts $f^{-1}(g^{-1}(\Re))$ is IT_i-open set in X. But $(g \circ f)^{-1}(\Re) = f^{-1}(g^{-1}(\Re))$ is IT_i-open set in X. Hence, g ∘ f: $(X, IT_i, IT_j) \to (Z, I\gamma_i, I\gamma_j)$ is strongly (i, j)- Ipre* cts.

b) Let Q be any (i, j)- Ipre* open set in Z. Since g is strongly (i, j)- Ipre* cts, $g^{-1}(\Re)$ is IT_i-open set in Y. Also, f is (i, j)- Ipre* continuous $f^{-1}(g^{-1}(\Re))$ is (i, j)- Ipre* open set in X. But $(g \circ f)^{-1}(\Re) = f^{-1}(g^{-1}(\Re))$ is (i, j)- Ipre* open set in X. Hence, g ∘ f: $(X, IT_i, IT_j) \to (Z, I\gamma_i, I\gamma_j)$ is (i, j)- Ipre* irresolute.

c) Let U be any $I\gamma_i$-open set in Z. Since g is (i, j)- Ipre* continuous, $g^{-1}(U)$ is (i, j)- Ipre* open set in Y. Also, f is strongly (i, j)- Ipre* cts $f^{-1}(g^{-1}(U))$ is IT_i-open set in X. But $(g \circ f)^{-1}(U) = f^{-1}(g^{-1}(U))$ is IT_i-open set in X. Hence, g ∘ f: $(X, IT_i, IT_j) \to (Z, I\gamma_i, I\gamma_j)$ is pairwise intuitionistic continuous.

d) Let V be any (i, j)- Ipre* open set in Z. Since g is strongly (i, j)- Ipre* cts, $g^{-1}(V)$ is $I\sigma_i$-open set in Y. Also, f is pairwise intuitionistic continuous $f^{-1}(g^{-1}(V))$ is IT_i-open set in X. But $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is IT_i-open set in X. Hence, g ∘ f: $(X, IT_i, IT_j) \to (Z, I\gamma_i, I\gamma_j)$ is strongly (i, j)- Ipre* cts.

Theorem 3.15 The composition of two strongly (i, j)- Ipre* cts maps is strongly (i, j)- Ipre* cts.

Proof: Let $\Re$ be any (i, j)- Ipre* open set in Z. Since g is strongly (i, j)- Ipre* cts, we get $g^{-1}(\Re)$ is $I\sigma_i$-open set in Y. By theorem 2.6 (i) $g^{-1}(\Re)$ is (i, j)- Ipre* open set in Y. Also, f is strongly (i, j)- Ipre* cts $f^{-1}(g^{-1}(\Re)) = (g \circ f)^{-1}(\Re)$ is IT_i-open set in X. Hence, g ∘ f is strongly (i, j)- Ipre* cts.

Definition 3.16 A function f: $(X, IT_i, IT_j) \to (Y, I\sigma_i, I\sigma_j)$ is called a Strongly (i, j)- Intuitionistic Semi* Continuous (strongly (i, j)- Isemi* cts) functions if the inverse image of each (i, j)- Isemi* open set in $(Y, I\sigma_i, I\sigma_j)$ is IT_i-open set in (X, IT_i, IT_j).

Equivalently, f: $(X, IT_i, IT_j) \to (Y, I\sigma_i, I\sigma_j)$ is called strongly (i, j)- Isemi* cts function if $f^{-1}(V) \in IT_i$-OS in (X, IT_i, IT_j) for each (i, j)- Isemi* open set V in $(Y, I\sigma_i, I\sigma_j)$.

Example 3.17 Let X = {s, t}, IT_i = {X, ∅, <X, ∅, ∅>, <X, {s}, ∅>, <X, ∅, {t}>, <X, {t}, ∅>}, IT_j = {X, ∅, <X, ∅, {s}>, <X, {s}, ∅>, <X, {t}, {s}>} and Y = {s, t}, $I\sigma_i$ = {Y, ∅, <Y, ∅, ∅>, <Y, {s}, ∅>, <Y, {t}, ∅>, <Y, ∅, {t}>}, $I\sigma_j$ = {Y, ∅, <Y, ∅, ∅>, <Y, {s}, ∅>, <Y, ∅, {t}>, <Y, {s}, {t}>}. Here, (i, j)- IS*O (Y) = {Y, ∅, <Y, ∅, ∅>, <Y, {s}, ∅>, <Y, ∅, {t}>, <Y, {t}, ∅>}. Let f: $(X, IT_i, IT_j) \to (Y, I\sigma_i, I\sigma_j)$ be defined as f(s) = s, f(t) = t. Clearly, f is strongly (i, j)- Isemi* cts.

Definition 3.18 A function f: $(X, IT_i, IT_j) \to (Y, I\sigma_i, I\sigma_j)$ is called a Strongly (i, j)- Intuitionistic α* Continuous (strongly (i, j)- Iα* cts) functions if the inverse image of each (i, j)- Iα* open set in $(Y, I\sigma_i, I\sigma_j)$ is IT_i-open set in (X, IT_i, IT_j).

Equivalently, f: $(X, IT_i, IT_j) \rightarrow (Y, I\sigma_i, I\sigma_j)$ is called strongly (i, j)- Iα* cts function if $f^{-1}(V) \in IT_i$-open set in (X, IT_i, IT_j) for each (i, j)- Iα*open set V in $(Y, I\sigma_i, I\sigma_j)$.

Example 3.19 Let X = {s, t}, IT_i = {X, ∅, <X, ∅, ∅>, <X, {t}, ∅>, <X, ∅, {s}>, <X, {t}, {s}>}, IT_j = {X, ∅, <X, ∅, {s}>, <X, {s}, ∅>} and Y = {s, t}, $I\sigma_i$ = {Y, ∅, <Y, ∅, ∅>, <Y, {t}, ∅>, <Y, {t}, {s}>}, $I\sigma_j$ = {Y, ∅, <Y, {s}, ∅>, <Y, ∅, {t}>}. Here, (i, j)- Iα*O (Y) = {Y, ∅, <Y, ∅, ∅>, <Y, ∅, {s}>, <Y, {t}, ∅>, <Y, {t}, {s}>}. Define f: $(X, IT_i, IT_j) \rightarrow (Y, I\sigma_i, I\sigma_j)$ by f(s) = s, f(t) = t. Clearly, f is strongly (i, j)- Iα* cts.

Remark 3.20 The following example shows that the strongly (i, j)- Isemi* cts function and strongly (i, j)- Ipre* cts function are independent.

Example 3.21 Let X = {s, t}, IT_i = {X, ∅, <X, ∅, ∅>, <X, {s}, ∅>, <X, ∅, {t}>, <X, {t}, ∅>}, IT_j = {X, ∅, <X, ∅, {s}>, <X, {s}, ∅>, <X, {t}, {s}>} and Y = {s, t}, $I\sigma_i$ = {Y, ∅, <Y, ∅, ∅>, <Y, {s}, ∅>, <Y, {t}, ∅>, <Y, ∅, {t}>}, $I\sigma_j$ = {Y, ∅, <Y, ∅, ∅>, <Y, {s}, ∅>, <Y, ∅, {t}>, <Y, {s}, {t}>}. Here, (i, j)- IP*O (Y) = {Y, ∅, <Y, ∅, ∅>, <Y, ∅, {t}>, <Y, {s}, ∅>, <Y, {t}, ∅>, <Y, {s}, {t}>} & (i, j)- IS*O (Y) = {Y, ∅, <Y, ∅, ∅>, <Y, {s}, ∅>, <Y, ∅, {t}>, <Y, {t}, ∅>}. Define f: $(X, IT_i, IT_j) \rightarrow (Y, I\sigma_i, I\sigma_j)$ by f(s) = s, f(t) = t. Here, the inverse image of each (i, j)- Isemi* open set in Y is IT_i-open set in X. Clearly, f is strongly (i, j)- Isemi* cts. But f^{-1}(<Y, {s}, {t}>) = <X, {s}, {t}> ∉ IT_i-open set in X. Hence, f is not strongly (i, j)- Ipre* cts.

Example 3.22 Let X = {a, b, c}, IT_i = {X, ∅, <X, ∅, {b, c}>, <X, {a, c}, ∅>}, IT_j = {X, ∅, <X, {a}, {b, c}>, <X, {b, c}, {a}>} and Y = {a, b, c}, $I\sigma_i$ = {Y, ∅, <Y, {a, c}, ∅>, <Y, ∅, {b, c}>}, $I\sigma_j$ = {Y, ∅, <Y, ∅, {a, c}>, <Y, {a, b}, {c}>}. Here, (i, j)- IP*O (Y) = {Y, ∅, <Y, {a, c}, ∅>, <Y, ∅, {b, c}>} & (i, j)- IS*O (Y) = {Y, ∅, <Y, ∅, {b}>, <Y, {a, c}, ∅>, <Y, ∅, {b, c}>}. Define f: $(X, IT_i, IT_j) \rightarrow (Y, I\sigma_i, I\sigma_j)$ by f(s) = s, f(t) = t. Here, the inverse image of each (i, j)- Ipre* open set in Y is IT_i-open set in X. Clearly, f is strongly (i, j)- Ipre* cts. But f^{-1}(<Y, ∅, {b}>) = <X, ∅, {b}> ∉ IT_i-open set in X. Hence, f is not strongly (i, j)- Isemi* cts.

Remark 3.23 The following example shows that the strongly (i, j)- Iα* cts function and strongly (i, j)- Ipre* cts function are independent.

Example 3.24 Let X = {s, t}, IT_i = {X, ∅, <X, ∅, ∅>, <X, {t}, ∅>, <X, ∅, {s}>, <X, {t}, {s}>}, IT_j = {X, ∅, <X, ∅, {s}>, <X, {s}, ∅>} and Y = {s, t}, $I\sigma_i$ = {Y, ∅, <Y, ∅, ∅>, <Y, {t}, ∅>, <Y, {t}, {s}>}, $I\sigma_j$ = {Y, ∅, <Y, {s}, ∅>, <Y, ∅, {t}>}. Here, (i, j)- IP*O (Y) = {Y, ∅, <Y, ∅, ∅>, <Y, ∅, {t}>, <Y, {s}, ∅>, <Y, {t}, ∅>, <Y, {t}, {s}>, <Y, {s}, {t}>} & (i, j)- Iα*O (Y) = {Y, ∅, <Y, ∅, ∅>, <Y, ∅, {s}>, <Y, {t}, ∅>, <Y, {t}, {s}>}. Define f: $(X, IT_i, IT_j) \rightarrow (Y, I\sigma_i, I\sigma_j)$ by f(s) = s, f(t) = t. Here, the inverse image of each (i, j)- Iα* open set in Y is IT_i-open set in X. Clearly, f is strongly (i, j)- Iα* cts. But f^{-1}(<Y, {s}, {t}>) = <X, {s}, {t}>, f^{-1}(<Y, ∅, {t}>) = <X, ∅, {t}>, f^{-1}(<Y, {s}, ∅>) = <X, {s}, ∅> which is not IT_i-open set in X. Hence, f is not strongly (i, j)- Ipre* cts.

Example 3.25 Let X = {s, t}, IT_i = {X, ∅, <X, ∅, ∅>, <X, {s}, ∅>, <X, ∅, {t}>, <X, {t}, ∅>}, IT_j = {X, ∅, <X, ∅, {s}>, <X, {t}, {s}>} and Y = {s, t}, $I\sigma_i$ = {Y, ∅, <Y, {s}, ∅>, <Y, ∅, {t}>}, $I\sigma_j$ = {Y, ∅, <Y, {t}, {s}>}. Here, (i, j)- IP*O (Y) = {Y, ∅, <Y, {s}, ∅>, <Y, ∅, {t}>} & (i, j)- Iα*O (Y) = {Y, ∅, <Y, ∅, {t}>, <Y, {s}, ∅>, <Y, {s}, {t}>}. Define f: $(X, IT_i, IT_j) \rightarrow (Y, I\sigma_i, I\sigma_j)$ by f(s) = s, f(t) = t. Here, the inverse image of each (i, j)- Ipre* open set in Y is IT_i-open set in X. Clearly, f is strongly (i, j)- Ipre* cts. But f^{-1}(<Y, {s}, {t}>) = <X, {s}, {t}> ∉ IT_i-open set in X. Hence, f is not strongly (i, j)- Iα* cts.

4. CONCLUSION

This study underscores the significance of Strongly (i, j)- Intuitionistic Pre* continuous functions in bi-Intuitionistic Topological Spaces. Also, further we conclude that the concept of strongly (i, j)- Ipre* cts and strongly (i, j)- Iα* cts, strongly (i, j)- Isemi* cts and strongly (i, j)- Ipre*cts are independent of each other. We intend to conduct research in the future on Perfectly (i, j)- Intuitionistic Pre* continuous, totally (i, j)- Intuitionistic Pre* continuous, (i, j)- Ipre* open maps and (i, j)- Ipre* closed maps and so on.

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